Trigonometric Functions

Case Study Based Questions

Read the following passages and answer the questions that follow:

1. Nitish is playing with a Pinwheel toy which he bought from a village fair. He noticed that the pinwheel toy revolves as fast as he blows it. Consider the Pinwheel toy that makes 360 revolutions per minute.

(A) Find the number of revolutions made by Pinwheel toy in 120 second.
(B) Find the number of revolutions made by Pinwheel toy in 1 sec and angle made by Pinwheel toy (in degree) in 6 revolutions.
(C) Find the radius of the circle in which a central angle of 60° intercepts an arc of length 37.4 cm. (Use $\pi = \frac{22}{7}$).

Ans. (A) Since the number of revolutions made by
Pinwheel toy in 1 minute = 360
And 1 min 60 seconds
So, the number of revolution made by Pinwheel toy in 60 seconds = 360
The number of revolution made by Pinwheel

$$\text{toy in 1 second} = \frac{360}{60}$$

Number of revolutions made by Pinwheel

$$\text{toy in 120 seconds} = \frac{360 \times 120}{60} = 720$$

(B) The number of revolution made by Pinwheel

toy in 1 seconds = $\frac{360}{60} = 6$

Since, angle made by Pinwheel toy in 1 revolutions = 360°.

Thus, angle made by Pinwheel toy in 6 revolutions 360° x 6 = 2160°

(C) Given,

Length of the arc = l = 37.4 cm

$$\text{Central angle} = \theta = 60° = \frac{60 \times \pi}{180} \text{ radian}$$

$$= \frac{\pi}{3} \text{ radians}$$

We know that,

$$r = \frac{l}{\theta}$$

$$= (37.4) \times \left(\frac{\pi}{3}\right)$$

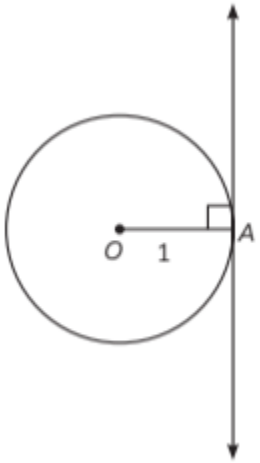
$$= \frac{(37.4)}{\left(\frac{22}{7 \times 3}\right)}$$

$$= 35.7 \text{ cm}$$

Hence, the radius of the circle is 35.7 cm.

2. Consider a unit circle with centre O. Let A be any point on the circle. Consider O A as the initial side of an angle. Then the length of an arc of the circle will give the radian measure of the angle which the arc will subtend at the centre of the circle.

A circle subtends an angle at the centre whose radian measure is 2 π and its degree measure is 360°



(A) The radian measure of 240° is:

(a) $\frac{4\pi}{3}$ (b) $\frac{2\pi}{3}$

(c) $\frac{5\pi}{3}$ (d) $\frac{\pi}{3}$

(B) A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?

(a) 6π

(b) 4π

(c) 3π

(d) 12π

(C) The degree measure of 1.2 radian is:

(a) 68°

(b) 68°43'37.8"

(c) 68°45'36"

(d) 58°46'27"

(D) The radius of the circle in which a central angle of 45° intercepts an arc of 132 cm, is:

(Use π) = $\frac{22}{7}$

(a) 168 cm

(b) 50 cm

(c) 160 cm

(d) 148 cm

(E) The minute hand of a watch is 35 cm long. How far does it move in 9 minutes?

(a) 15 cm

(b) 30 cm

(c) 46 cm

(d) 33 cm

Ans. (A)

(a) $\frac{4\pi}{3}$

Explanation: As we know that

180° = radian

$$1° = \frac{\pi}{180°} \text{ radian}$$

$$\text{Radian measure of } 240° = 240 \times \frac{\pi}{180°}$$

$$= \frac{4\pi}{3}$$

(B) (d) 12π

Explanation: Given that a wheel makes 360 revolutions in 1 minute. i.e., a wheel makes 360 revolutions in 60 seconds.

$$\therefore \text{ In 1 second, no. of revolutions} = \frac{360}{60}$$

= 6 revolutions

In 1 revolution, the angle made by the wheel

= 360°

:- Angle made by the wheel in 6 revolutions

= 6 x 360°

= 2160°

Radian made in 6 revolutions

$$= 2160° \times \frac{\pi}{180°}$$

$$= 12\pi$$

(C) (b) 68°43'37.8"

Explanation: As we know that,

180° = π radian

$$1 \text{ radian} = \frac{180°}{\pi}$$

$$(1.2) = \left(1.2 \times \frac{180}{\pi}\right)$$

$$= 1.2 \times \frac{180}{22} \times 7$$

= 68.7272°

= 68° (0.7272 x 60)'

= 68° 43′(0.63 x 60)"

= 68° 43' 37.8"

(D) (a) 168 cm

Explanation: We have,

$$l = 132 \text{ cm and } \theta - 45° = 45 \times \frac{\pi}{180} = \frac{\pi}{4}$$

$$\text{Now, } \theta = \frac{l}{r} = \frac{132}{r}$$

$$\frac{\pi}{4} = \frac{132}{r}$$

$$\Rightarrow \quad r = \frac{132 \times 4}{\pi}$$

$$\Rightarrow \quad r = \frac{132 \times 4}{\frac{22}{7}} = 168 \text{ cm}$$

(E) (d) 33 cm

Explanation: The angle made by minute hand in 9 minutes = (9 x 6)°

$$= 54° = 54° \times \frac{\pi}{180} = \frac{3\pi}{10}$$

$$\therefore \quad \theta = \frac{l}{r}$$

$$\Rightarrow \quad \frac{3\pi}{10} = \frac{l}{35}$$

$$\Rightarrow \quad l = \frac{35 \times 3\pi}{10} = \frac{21\pi}{2}$$

$$\Rightarrow \quad l = \frac{21}{2} \times \frac{22}{7} = 33 \text{ cm}$$

3. Sudhir who is a student of class XI got a Maths assignment from his class teacher. He did all the questions except a few. If the value

$$of \ \sin x = \frac{3}{5} \ and \ \cos y = -\frac{12}{13}, \ where \ x \ and \ y$$

both lie in the second quadrant, then help Sudhir in solving these questions.

(A) What will be the value of cos x?

(a) $\frac{4}{5}$ (b) $\frac{-3}{5}$

(c) $\frac{-4}{5}$ (d) $\frac{3}{5}$

(B) What will be the value of sin y?

(a) $\frac{5}{12}$ (b) $\frac{-12}{13}$

(c) $\frac{-5}{13}$ (d) $\frac{5}{13}$

(C) Which of the following options is correct?

(a) sin (x - y) = sin x cos y + cos x sin y

(b) sin (x + y) = cos x sin y - sin x cos y

(c) sin (x + y) = sin x cos y + cos x sin y

(d) sin(x - y) = sin x sin y - cos x cos y

(D) The value of sin (x + y) is:

(a) $\frac{-56}{65}$ (b) $\frac{56}{65}$

(c) $\frac{55}{67}$ (d) $\frac{-55}{67}$

(E) The value of sin 75° is:

(a) $\frac{1-\sqrt{3}}{\sqrt{2}}$ (b) $\frac{1+\sqrt{3}}{2\sqrt{2}}$

(c) $\frac{1-\sqrt{3}}{2\sqrt{2}}$ (d) $\frac{1+\sqrt{3}}{2}$

Ans. (A)

(c) $\frac{-4}{5}$

Explanation: Given, $\sin x = \frac{3}{5}$

As we know that

$$\cos^2 x = 1 - \sin^2 x$$

$$= 1 - \left(\frac{3}{5}\right)^2$$

$$= 1 - \frac{9}{25}$$

$$= \frac{25-9}{25} = \frac{16}{25}$$

Thus, $\cos x = \pm\frac{4}{5}$

Since x lies in second quadrant

$\therefore$ $\cos x$ is negative

$\therefore$ $\cos x = -\frac{4}{5}$

(B) (d) $\frac{5}{13}$

Explanation: Given,

$$\cos y = \frac{-12}{13}$$

As we know that

$$\sin^2 y = 1 - \cos^2 y$$

$$= 1 - \frac{144}{169} = \frac{25}{169} = \pm\frac{5}{13}$$

Since, y lies in second quadrant

$\therefore$ $\sin y$ is positive

$\therefore$ $\sin y = \frac{5}{13}$

(C) (c) $\sin(x + y) = \sin x \cos y + \cos x \sin y$

Explanation: Trigonometric function of compound angle

$\sin(x + y) = \sin x \cos y + \cos x \sin y$

(D) (a) $\frac{-56}{65}$

(D)

Explanation:

As we know that

sin (x + y) = sin x cos y + cos x sin y

$$\sin(x+y)=\left(\frac{3}{5}\right)\times\left(-\frac{12}{13}\right)+\left(-\frac{4}{5}\right)\times\left(\frac{5}{13}\right)$$

$$=-\frac{36}{65}-\frac{20}{65}=-\frac{56}{65}$$

(E)

(b) $\frac{1+\sqrt{3}}{2\sqrt{2}}$

Explanation: Given,

$$\sin 75°=\sin(30°+45°)$$

$$=\sin 30°\cos 45°+\cos 30°\sin 45°$$

$$=\frac{1}{2}\cdot\frac{1}{\sqrt{2}}+\frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}}=\frac{1+\sqrt{3}}{2\sqrt{2}}$$

4.

Consider $\sin A=\frac{4}{5}$ **and** $\cos B=\frac{5}{13}$ **where**

$0<A,B<\frac{\pi}{2}$.

(A) Find the value of cos A + sin B.

(B) Find value of cos (A + B).

(C) Find the value of sin (A - B).

Ans.

(A) Given, $\sin A = \frac{4}{5}, 0 < A < \frac{\pi}{2}$

$\therefore \quad \cos A = \sqrt{1 - \sin^2 A}$

[$\because$ A lies in 1st quadrant]

$$= \sqrt{1 - \left(\frac{4}{5}\right)^2} = \sqrt{1 - \frac{16}{25}}$$

$\Rightarrow \quad \cos A = \sqrt{\frac{9}{25}} = \frac{3}{5}$

And $\quad \cos B = \frac{5}{13}, 0 < B < \frac{\pi}{2}$

$\therefore \quad \sin B = \sqrt{1 - \cos^2 B}$

[$\because$ B lies in 1st quadrant]

$$= \sqrt{1 - \left(\frac{5}{13}\right)^2} = \sqrt{1 - \frac{25}{169}}$$

$\Rightarrow \quad \sin B = \sqrt{\frac{144}{169}} = \frac{12}{13}$

$$\cos A + \sin B = \frac{3}{5} + \frac{12}{13} = \frac{39 + 60}{65} = \frac{99}{65}$$

(B) $\cos (A+B) = \cos A \cos B - \sin A \sin B$

$$= \frac{3}{5} \times \frac{5}{13} - \frac{4}{5} \times \frac{12}{13}$$

$$= \frac{15 - 48}{65} = \frac{-33}{65}$$

(C) $\sin (A - B) = \sin A \cos B - \cos A \sin B$

$$= \frac{4}{5} \times \frac{5}{13} - \frac{3}{5} \times \frac{12}{13}$$

$$= \frac{20 - 36}{65} = \frac{-16}{65}$$